Second stage of Israeli students competition, 2015.

Duration: 4 hours

1. Find such $x>0$, for which $\int_0^x \frac{dt}{t^{1+\ln t}} = \int_x^\infty \frac{dt}{t^{1+\ln t}}$.

2. N people must travel from one end of the road to another. The length of the road is L. They have K bicycles (K < N). The velocity of walking man is v_1, and the velocity of a bicycle is v_2 (obviously, $v_1 < v_2$). How much time is required?

3. A unit cube in 4-dimensional Euclidean space contains a 3-dimensional Euclidean ball of radius R. What is the greatest possible value of R?

4. The sequence $\{a_n\}$ is defined by recurrent formula $a_{n+1} = a_n + \sqrt{1+a_n^2}$, and $a_1 = 1$. Compute $\lim_{n\to\infty} \frac{2^n}{a_n}$.

5. Polynomials $P(x)$ and $Q(x)$ of odd degree are such that for each integer x there is an integer y such that $P(x) = Q(y)$. Prove that there exists a polynomial R, such that $P(x) = Q(R(x))$ for each x.

6. For given 2×2 matrices A, B there is only finite number n of 2×2 matrices X such that $X^2 + AX + B = 0$. Find the maximal possible value of n. (All matrices in this questions have complex entries.)

Good luck!