

# First stage of Israeli students competition, 2015.

Duration: 4 hours

אנחנו מבקשים לכתוב את הפתרונות באנגלית, אבל זה בסדר מדי פעם להשתמש בעברית כאשר אתם מתקשים לנסח הוכחה מורכבת כלשהי. את השאלון ניתן לקחת איתכם.

1. Compute  $\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{n}}}$ .

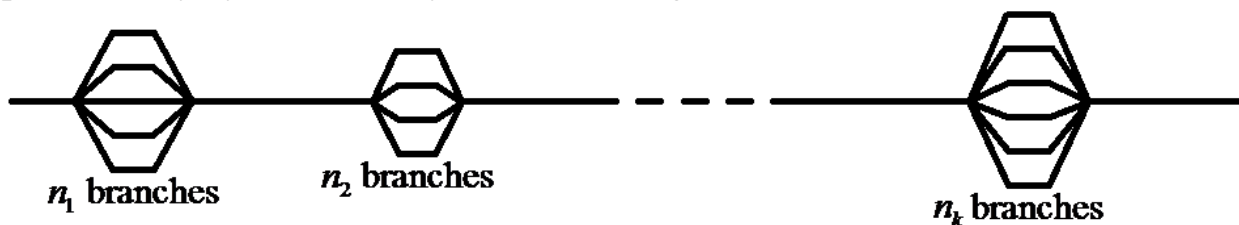
2. Compute  $\det \begin{pmatrix} 1 & 2 & 2 & 3 \\ 2 & 2 & 3 & 3 \\ 2 & 3 & 3 & 4 \\ 3 & 3 & 4 & 4 \end{pmatrix}$ .

3. Let  $A, B, C, D$  be points in 3-dimensional Euclidean space not in the same plane, such that the plane  $ACB$  is orthogonal to the plane  $ACD$ , and the plane  $ABD$  is orthogonal to the plane  $CBD$ . Prove that  $\frac{\cos(\angle ACB)}{\cos(\angle ADB)} = \frac{\cos(\angle CBD)}{\cos(\angle CAD)}$ .

4. A finite number of polyhedrons of positive volume in 3-dimensional Euclidean space is given. Prove that one can mark a finite number of points in the same Euclidean space, so that strictly inside any two of given polyhedrons of equal volume, there will be the same number of marked points, and every given polyhedron will contain at least one point.

5. Prove that sum of digits of  $2^{4^{1000001}}$  is greater than 1000000.

6.  $M$  cars move from left to right on a narrow road (they can't overtake each other, and cannot go backwards, all cars start at the left end and arrive to the right end). In  $k$  places, the road is split in parallel routes: first in  $n_1$  parallel branches which are merged again, then in  $n_2$  parallel branches, etc. Each branch is long enough to contain any amount of cars. For which  $M$  is it possible to reorder the cars in any possible way by the time they arrive to the right end of the road?



Good luck!