

First stage of Israeli students competition, 2019.

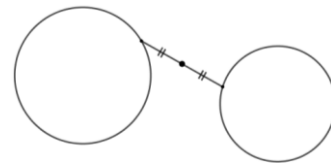
Please try to write your solutions in English.

Duration: 4 hours

את השאלון ניתן לקחת איתך בסיום התחרות.

1. Let A, B be two orthogonal $n \times n$ matrices with real entries. What is the maximal possible value of $\det(A + B)$?

2. Two circles are given in plane: circle α of radius a , and circle β of radius b . Consider all midpoints of intervals, connecting a point on α and a point on β .



What is the area of the set of all such midpoints?

3. Compute $\lim_{n \rightarrow \infty} \frac{\sum_{k=1}^{2n} (n - |n - k|) \cdot \sqrt[n]{e^k}}{n^2}$.

4. Consider a graph whose vertices are all the binary strings of length 4, where two strings are connected if they differ at exactly one digit. For example $(0,1,1,0)$ is connected to $(0,1,0,0)$. Is it possible to find two Hamiltonian cycles, which don't have any common edge?

Reminder. A *Hamiltonian cycle* (מעגל המילטון) is a path which visits each vertex exactly once and comes back to the original vertex.

5. Let $F_1, F_2, F_3, F_4, \dots$ be an infinite sequence of pairwise disjoint

non-empty closed subsets of \mathbb{R} . Might it be that $\bigcup_{n=1}^{\infty} F_n = \mathbb{R}$?

6. Let $0 < a_0 < a_1 < \dots < a_n$. Prove that the equation

$$a_0 + a_1 \cos(x) + a_2 \cos(2x) + \dots + a_n \cos(nx) = 0$$

has precisely n solutions in the interval $[0, \pi]$.

Good luck!