

הרמת אקספוננט:

$$v_p(a^n - b^n) = v_p(a - b) + v_p(n), \quad p \neq 2$$

$$v_2(a^n - b^n) = v_2(a - b) + v_2(a + b) + v_2(n) + 1, \quad 2 | n$$

$$v_2(a^n - b^n) = v_2(a - b) + v_2(n) \quad 4 | a - b$$

דוגמאות:

$$\Phi_1 = a - b$$

$$\Phi_2 = a + b$$

$$\Phi_3 = a^2 + ab + b^2$$

$$\Phi_4 = a^2 + b^2$$

$$\Phi_5 = a^4 + a^3b + a^2b^2 + ab^3 + b^4$$

$$\Phi_6 = a^2 - ab + b^2$$

$$\Phi_7 = a^6 + a^5b + a^4b^2 + \dots + ab^5 + b^6$$

$$\Phi_8 = a^4 + b^4$$

$$\Phi_9 = a^6 + a^3b^3 + b^6$$

$$\Phi_{10} = a^4 - a^3b + a^2b^2 - ab^3 + b^4$$

פולינומים ציקלוטומיים:

$$\Phi_n(a, b) = \prod_{\text{ord } \xi = n} (a - \xi b)$$

$$\deg(\Phi_n) = \phi(n)$$

$$a^n - b^n = \prod_{d|n} \Phi_d(a, b)$$

$$\Phi_n(a, b) = \prod_{d|n} \left(a^{n/d} - b^{n/d} \right)^{\mu(d)}$$

$$\Phi_{p^k \cdot n}(a, b) = \begin{cases} \Phi_n(a^{p^k}, b^{p^k}), & p | n \\ \frac{\Phi_n(a^{p^k}, b^{p^k})}{\Phi_n(a^{p^{k-1}}, b^{p^{k-1}})}, & p \nmid n \end{cases}$$

אי-שוויון חשוב:

$$|a - b|^{\phi(n)} \leq |\Phi_n(a, b)| \leq |a + b|^{\phi(n)}$$