First stage of Israeli students competition, 2018.

Please try to write your solutions in English.

Duration: 4 hours

את השאלון ניתן לקחת איתך בסיום התחרות.

1. Compute $\det\begin{pmatrix} 1 & 2 & 2 & 2 & 2 \\ 1 & 3 & 4 & 4 & 4 \\ 1 & 3 & 5 & 6 & 6 \\ 1 & 3 & 5 & 7 & 8 \\ 1 & 3 & 5 & 7 & 9 \end{pmatrix}$.

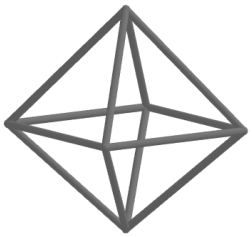
2. Consider the polynomial $p(x)=(x+1)^{90}+(x-1)^{90}$.

(a) Find all complex roots of p.

(b) Compute $\tan^2(1^\circ)+\tan^2(3^\circ)+\tan^2(5^\circ)+\tan^2(7^\circ)+\ldots+\tan^2(89^\circ)$.

3. Prove that $\lim\limits_{t\to 1-}\left(\sqrt[3]{1-t}\cdot\left(1+t+t^8+t^{27}+\ldots+t^{n^3}+\ldots\right)\right)=\int\limits_0^\infty e^{-x^3}dx$.

4. A wolf hunts a rabbit on a space station, consisting of 12 corridors which form a regular octahedron. The wolf can see the rabbit only if they are in the same corridor. The wolf is 2.5 times faster than the rabbit. Can the wolf devise a strategy to catch the rabbit in any case?



5. A two-dimensional square with side of length 10 is contained in a unit cube of dimension n. What is the least possible n?

Good luck!