Second Stage of Israeli Olympiad for University Students.

1. On a plane N lines in **general position** are chosen. General position means, that no two lines are parallel, no three lines are concurrent, and neither three intersection points belong to the same unchosen line.
An additional line will be called good, if it doesn't pass through any intersection points of chosen lines. Additional lines will be considered equivalent, if one can be obtained from the other by continuous motion, such that all intermediate lines are good as well. Find the number of equivalence classes of good lines.

2. Evaluate the integral $\int_0^\infty \left(e^{-2x}-e^{-x}\right)\frac{dx}{x}$.

3. Let $a,b,c,d,v\in\mathbb{R}^3$. Show that
$\langle[a,b],v\rangle\cdot\langle[c,d],v\rangle-\langle[a,c],v\rangle\cdot\langle[b,d],v\rangle+\langle[a,d],v\rangle\cdot\langle[b,c],v\rangle=0$.
Here $\langle,\rangle$ is scalar product and $[,]$ is vector product.

4. Let $\alpha>0$ be an irrational number, $\beta\in(0,1)$. Denote:

$$Q(m)=\min_{1\le n\le m}\{n\alpha\}$$

$$R(m)=\min_{1\le n\le m}\{\beta-n\alpha\}$$

(where *n* runs over integer numbers between 1 and *m*).
Prove that there is infinite number of values m, such that $Q(m)>R(m)$.

5. What is the maximal possible area of an ellipse, which is contained in the upper half of a unit circle?

6. Let p be an odd prime, and let $GL_2(\mathbb{F}_p)$ be the set of all invertible 2×2 matrices over the field with p elements. A partition of $GL_2(\mathbb{F}_p)$ will be called **nice** if every two matrices belonging to the same set commute. Determine the minimal number of sets in the nice partition.

Good luck!