First stage of Israeli students competition, 2012.

Duration – 4 hours.

1. Find the determinants of the following matrixes (the answer might depend on *a*)

a. $\begin{pmatrix} 1+a^2 & a & 0 & 0 & 0 & 0 \\ a & 1+a^2 & a & 0 & 0 & 0 \\ 0 & a & 1+a^2 & a & 0 & 0 \\ 0 & 0 & a & 1+a^2 & a & 0 \\ 0 & 0 & 0 & a & 1+a^2 & a \\ 0 & 0 & 0 & 0 & a & 1+a^2 \end{pmatrix}$ b. $\begin{pmatrix} 1+a^2+a^4 & a+a^3 & a^2 & 0 & 0 \\ a+a^3 & 1+a^2+a^4 & a+a^3 & a^2 & 0 \\ a^2 & a+a^3 & 1+a^2+a^4 & a+a^3 & a^2 \\ 0 & a^2 & a+a^3 & 1+a^2+a^4 & a+a^3 \\ 0 & 0 & a^2 & a+a^3 & 1+a^2+a^4 \end{pmatrix}$

2. A function f has the following property $\int_{-1}^{1} f(x)x^n dx = 0$ for *n*=0, 1, …, 5771.

Prove that f has at least 5772 roots in the interval $[-1,1]$.

3. ABC is a triangle. Consider all intervals PQ, such that P is on AB, Q is on BC, and PQ divides ABC into two parts of equal area. The union of all intervals PQ will be denoted by *U*. Compute $\frac{Area(U)}{Area(\text{ABC})}$.

4. A sequence $\{a_n\}$ is such that $a_n > 0, \forall n$ and $a_n \underset{n\to\infty}{\to} 0$, however, $\sum_{n=1}^{\infty} a_n = \infty$.

Denote $s_n = \sum_{k=1}^{n} a_k$. Prove that $\sum_{k=1}^{\infty} \frac{a_k}{s_k^2}$ always converges.

5. Prove that there is an infinite quantity of natural numbers *n* such that *n* appears in the end of the decimal representation of 2^n (for example: 2^{36} ends with 36).