First stage of Israeli students competition, 2009-2010.

Duration: 4 hours

1. Compute: a. $\det\begin{pmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{pmatrix}$ b. $\det\begin{pmatrix} 1 & 4 & 9 & 16 \\ 4 & 9 & 16 & 25 \\ 9 & 16 & 25 & 36 \\ 16 & 25 & 36 & 49 \end{pmatrix}$

2. a. How many planes are required to cut all the edges of a cube?
b. How many planes are required to cut each edge of a cube twice?
Remark. Edges of a polytope (מקצועות של פאון) are the intervals which are the sides of its faces. We say that a plane cuts an interval if the plane contains precisely one internal point of that interval.

3. Find all continuously differentiable functions $f:\mathbb{R}\to\mathbb{R}$ satisfying:

$$f(x+y)-f(x-y)=2y\cdot f'(x) \qquad \forall x,y\in\mathbb{R}.$$

4. The Department of Social Equality has 15 workers. In the beginning, each has a salary which is a positive integer number of NIS no greater than 10. Each year, the boss can raise the salaries of exactly 13 workers by 1 NIS simultaneously.
Workers are immortal, they never quit or retire; new workers are never accepted.
The boss wants to make the salaries of all workers in the Department equal.
a. Prove that it is possible.
b. In the worst case, how many years will it take?

5. A function *c* over the set of natural numbers is defined as follows:
$c(n)=$ 0 if there are even number of ones in binary representation of *n*,
1 otherwise.
A positive integer number *k* is given.
Let $l(N)$ be the number of integers *n* from 0 to *N*, such that $c(k+n)\neq c(n)$.
Prove that $\lim\limits_{N\to\infty}\dfrac{l(N)}{N}$ exists and belongs to $\left[\dfrac{1}{3},\dfrac{2}{3}\right]$.

Good luck!