Targil 9.

This targil is about eigenvalues.

1. Is it true for any real square matrix A that it is similar to A^T ?
(in other words, is it true that for any A there's invertible Q such that $A^T = QAQ^{-1}$).

2. A is real skew-symmetric matrix (meaning $A = -A^T$).
a. Prove that its eigenvalues are imaginary,
b. Can it have non-trivial Jordan cells (of size > 1)?

3. On a circle, *n* real numbers are written, not all of them are equal. Each second, all numbers change simultaneously: every number is replaced by the average of itself and its clockwise neighbor.
a. Prove that all numbers on the circle converge to the average of all numbers.
b. Prove it converges exponentially, in other words, the distance of a value at some point after *k* steps from its limit value is $O(\alpha^k)$ where is a real number in (0, 1).
c. Compute the value of that α (the minimal value for which that claim will hold for any starting configuration).

4. Prove: if the minimal polynomial of *A* is $(x-\lambda_1)^{n_1}(x-\lambda_2)^{n_2}\cdot\ldots\cdot(x-\lambda_k)^{n_k}$, then the minimal polynomial of $\begin{pmatrix} A & I \\ 0 & A \end{pmatrix}$ is $(x-\lambda_1)^{n_1+1}(x-\lambda_2)^{n_2+1}\cdot\ldots\cdot(x-\lambda_k)^{n_k+1}$.

5. Denote $A^{\#} = (I + A)^{-1}(I - A)$.
a. Show that $A^{\#\#} = A$.
b. Show that A is an orthogonal matrix iff $A^{\#}$ is skew-symmetric.