Targil 8.

This targil is about 3d geometry, but mostly about cubes, as you could have guessed by its number.

1. What is the radius of the largest planar disc inside the unit cube?

2. A box $\{0 \le x \le 1\ ,\ 0 \le y \le 1\ ,\ 0 \le z \le A\ \}$, where A is a positive real, is intersected by a family planes $\{x + y + z = n + \alpha\}$, where α is a real number, and n are all possible integer numbers. The intersections of this family of planes and the box gives a family of planar polygons.
a. Prove that sum of areas of these polygons does not depend on α.
b. Prove that mass center of all these polygons is the center of the box.

3. What is the greatest possible area of the orthogonal projection of the unit cube? (among all possible directions)

4. a. What is the greatest triangular planar section of a tetrahedron (not necessarily regular)?
b*. What is the greatest possible planar section of a tetrahedron?

5. a**. What is the greatest possible area of a planar section of a unit cube?
b**. Same question for the box $a \times b \times c$.