Targil 6.

This targil is inspired by SEEMOUS 2010.

1. a.* Question from the last SEEMOUS: given a real 2×2 matrix A, prove that there are two 2×2 real matrixes B and C such that $A = B^2 + C^2$ (this was the nicest problem in the competition).

b.** The natural generalization: for real matrixes $n \times n$, is it possible to represent each matrix as a sum of squares, and if yes, how many squares are required?

2. a. Prove that a real matrix which is sufficiently close to the unit matrix is a square of real matrix.
b.* Let D be a diagonal matrix with positive numbers at the diagonal. Prove that a matrix which is sufficiently close to D is a square of real matrix. (If you don't know what is "sufficiently close" stop complaining and invent a definition.)

3. Is it true that every complex $n \times n$ matrix is a square of a complex matrix?

4. A determinant of a 2×2 real matrix is positive. Is it true that this matrix is a square of a real matrix?

5. Is $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ a sum of two squares of real matrixes?