Targil 3.

Reminder: a set is convex, if for any two points inside the set, it contains the interval connecting these two points.

1. Consider a subset in $\mathbb{R}^K$. At each step, we add to our subset all the interior points of all intervals having both ends in that subset. The process is stopped when the set is already convex. What is the maximal possible number of steps in this process?

2. A family of N convex sets in $\mathbb{R}^K$ is given, $N > K$. Each $K + 1$ sets of the family have a common point. Prove that all sets have a common point.

3. Matrix S is square and has the following 3 properties:

(a) All entries are nonnegative.
(b) Sum of numbers in any row is 1.
(c) Sum of numbers in any column is 1.

Prove that this matrix is a weighted average of permutation matrixes.

4. Given k balls of radius 1 in $\mathbb{R}^3$ a point on the boundary of a ball is called "isolated" if it doesn't see any other ball (the balls are not transparent).
What is the area of the set of isolated points?

5. Consider a bounded convex shape of area S in plane with smooth boundary of length l. Find the area of R-neighborhood given R, S and l. (By R-neighborhood we mean the set of all points that have distance at most R from the original shape.)

6. We are given $N > 2$ circles of radius 1. Every straight line meets less than 3 circles. Their centers are O_1, O_2, … O_N. Prove that

a. $$\sum_{i<j} \frac{1}{O_i O_j} < \frac{n\pi}{4}$$

b.* $$\sum_{i<j} \frac{1}{O_i O_j} < \frac{(n-1)\pi}{4}$$