Targil 2.

(Questions about existence of weird analytical objects)

1. A $\{a_n\}$ is a decreasing sequence of positive numbers, $\sum_{n=1}^{\infty} a_n$ is a divergent series (תור מתבדר). Consider the sequence $\{b_n = \min(a_n, 1/n)\}$.

Can we claim that $\sum_{n=1}^{\infty} b_n$ diverges?

2.* Is it possible to construct $f:[0,1]\to\mathbb{R}$ which is continuous, monotone non-decreasing and satisfies the following two equations for every $x\in[0,1]$:

$$f(x/3) = f(x)/2$$
$$f(x) + f(1-x) = 1$$

3.* Derivative of a continuous function $f:\mathbb{R}\to\mathbb{R}$ at all rational points exists and equals zero. Is f necessarily a constant?

4. $\{a_n\}$ is a sequence of positive numbers, $\sum_{n=1}^{\infty} a_n = 1$. Prove that it is possible to insert into the open interval (0,1) a countable set of closed mutually disjoint intervals, indexed by positive integers, so that a closed interval number n is of length a_n.

5. $f:\mathbb{R}\to\mathbb{R}$ such that for any positive real x, y the sequence $f(x+ny)$, for $n\in\mathbb{N}$, tends to infinity.

a. Can we claim that that $f(x)\underset{x\to\infty}{\to}\infty$?

b.** Can we claim that $f(x)\underset{x\to\infty}{\to}\infty$, if it is given that f is continuous?