Targil 10.

This targil is about … well, I bet you'll guess.

1. Prove that for any $0 < p < 1$, and integers $m, n > 1$,

$$(1 - p^n)^m + (1 - (1 - p)^m)^n > 1.$$

2. The floor is tiled by $l \times l$ squares (so it looks like a lattice). You throw a needle of length n on the floor. What is the chance that when it falls, it won't cut the lines, but will be entirely within one of the squares?

3. For any natural numbers k, m, n compute the integral

$$\int_0^1 \left(\int_0^{1-y} y^k x^m (1 - x - y)^n \, dx \right) dy .$$

4.* **a.** Four aliens land on the surface of a spherical planet, randomly and independently (a chance for every alien to land inside a country is proportional to the area of this country). What is the probability that they all landed on the same hemisphere? In other words, what's the probability that there exists a plane through the planets center, such that all landing points are on the same side of that plane?
b. Ten aliens land on the planet (independently, proportional to surface area). What is the probability that they are on the same hemisphere?

5.** A unit cube is orthogonally projected on a random plane (the normal vector to the plane is distributed uniformly over the unit sphere).
What is the expectation of projection area?