Targil 1 - determinants.

1. All entries of a 10×10 matrix A belong to the interval [-1,1].
Can it happen that $\det A > 5770$?

2. Matrix B has zeroes on the main diagonal and ones at all other places.
a. Prove that B is non-degenerate.
b. Compute $\det B$.

3. a. Compute $u = \det \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 4 & 5 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 2^3 & 3^3 & 4^3 & 5^3 \end{pmatrix}$

b. Prove that $v = \det \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 4 & 5 \\ 2^4 & 3^4 & 4^4 & 5^4 \\ 2^7 & 3^7 & 4^7 & 5^7 \end{pmatrix}$ is divisible by u.

4*. The inverse and the determinant of the following matrix: $\begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \end{pmatrix}$.

5.** Consider an anti-symmetric ($A = -A^T$) matrix with integer coefficients.

Show that the determinant is a perfect square.