

Olympiad of Israel Mathematical Union

Selection of the team for IMC 2009

Please write your solutions in English

1. Denote A be number of ways to paint the cells of the 8×8 chessboard in 3 colors, so that no two adjacent cells are of the same color (by adjacent cells we mean cells having common side). Denote X the number of ways to write integer numbers in the cells of the chessboard, so that the number in the bottom left corner is 0, and the difference between numbers in any two adjacent cell is 1 (here by difference of x and y we mean $|x - y|$).

Express X via A .

2. Let $ABCD$ be a convex planar cyclic quadrilateral (מרובע קמור חסום) and P a point in space. Show that $PD^2 \cdot S_{ABC} + PB^2 \cdot S_{ACD} = PA^2 \cdot S_{BCD} + PC^2 \cdot S_{ABD}$.

3. It is given that $\sum_{i=1}^{\infty} x_i$ converges, and $\{x_i\}$ is a sequence of real numbers.

Can we claim that $\sum_{i=1}^{\infty} \sin(x_i)$ converges?

4. Suppose A is an $m \times n$ matrix and B is an $n \times m$ matrix. Prove that the set of nonzero eigenvalues of AB coincides with the set of nonzero eigenvalues of BA .

5. (a) Find a function defined on closed interval $[-1,1]$, which has only finite number of discontinuity point, such that its graph is invariant under rotation by the right angle around the origin.

(b) Prove that there is no function on open interval $(-1,1)$ which satisfies the same conditions.

Good luck!