

## Second stage of Israeli students competition, 2009.

1. Which is bigger:  $\operatorname{arctg}(e)$  or  $\frac{\pi}{4} + \frac{1}{2}$ ?

Calculator is not allowed.

2. Prove that  $\frac{1}{4} + \frac{1}{7} + \frac{1}{10} + \dots + \frac{1}{3n+1}$  is non-integer for any  $n$ .

3. A triangle is contained by an 11-dimensional unit cube inside  $\mathbb{R}^{11}$ . What is the maximal possible perimeter of that triangle?

4. Can a polynomial with rational coefficients have  $-\sqrt{2}$  as its minimal value?

5. Consider a shape consisting of a finite number of unit square cells. We try to cover a board of  $m \times n$  cells by equivalent copies of that shape, so that each cell of the board will be covered by the same number of layers.

Prove that it is impossible if and only if we can write a real number in each cell of the board, in such a way that the sum of all those numbers will be strictly negative, while a sum that can be covered by the given shape is strictly positive (wherever we place it on the board).

*Good luck!*