

## First stage of Israeli students competition, 2009.

1. Let  $C$  be a convex polygon and  $P$  a point inside it. Let  $N$  be number of vertices, such that an interval connecting  $P$  to the vertex divides the angle of  $C$  into two acute angles. Denote  $n$  number of sides of  $C$ , such that the foot of perpendicular from  $P$  to that side is strictly inside that side.

Proof that  $N = n$ .

2. Let  $A$  be a  $2 \times 2$  invertible matrix with real coefficients. One of its coefficients is 200. Can it happen that all the coefficients of matrices  $A^{-1}, A^2, A^3, A^4, \dots, A^{100}$  belong to the interval  $(-10, 10)$  ?

3. The sequence  $\{x_i\}$  is defined by the initial value  $x_0 \in [0,1]$  and recursive

formula 
$$x_{n+1} = \frac{1 - \sqrt{1 - x_n}}{2}.$$

Find  $\lim_{n \rightarrow \infty} (x_n \cdot 4^n)$ .

4. Two players play a game on the infinite chess-board. First player plays with 3 white pieces called sheep, and the second player plays with 3 black pieces, called wolves. They move in turn. In his move each player can move only one piece to an adjacent cell (having a common side with its previous cell). Sheep can be moved only horizontally. If a wolf and a sheep happen to be in the same cell, the wolf eats the sheep. Is it always possible for wolves to catch at least one sheep?

5. When in three-dimensional space the center of the ball of radius  $r$  goes along a circle of radius  $R$  (here  $R > r > 0$ ), the ball covers a three-dimensional body called torus. Compute the surface area of that torus as a function in  $r$  and  $R$ .

*Good luck!*