Group action.

For definition see http://en.wikipedia.org/wiki/Group_action

This targil contains a crucial error, but I won't say which problem.

1. (a) Assume that prime number p divides $x^{16} + 1$. Then p is of form $32k + 1$.
(b) Without using the general Dirichlet theorem or L-functions, prove that for any k there is infinite set of primes of form $kn + 1$.

2. (a) Pizza consists of p sectors (p is a prime number). Each triangle can be one of a types (onions, olives, mushrooms and so on). Compute the number of possible nonequivalent pizzas.
(b)* Let p be a prime number. Compute the quantity of subsets of p elements of a set $\{1, 2, 3, \ldots, 2p\}$ such that sum of subset elements is divisible by p.

3*. The order of group G is $2^k(2l+1)$, and the group has an element of order 2^k. Prove that elements of odd order form a subgroup.

4.** Prove that the group of rotations of dodecahedron is A_5 (the group of even permutations of 5 elements), and groups of all dodecahedron symmetries is S_5 (the group of all permutation of 5 elements).

5. Let p be a prime number.
(a) Show that group of order p^k has nontrivial center (in other words, it has elements other than unit that commute with all the other elements).
(b) Show that any group of p^2 elements is commutative.
(c) Can a group of p^3 elements be non-commutative?