Targil 7 – discrete convolution.

1. Without computer or calculator, find the decimal representation of $\frac{1}{81}$.

2. **(a)** Is it true, that for each polynomial with real coefficients p(x) there exists a polynomial q(x), such that q(x^3) is divisible by p(x) ?
(b) A point will be called ***even*** if both coordinates are integer even numbers. Function $f:\mathbb{Z}^2 \to \mathbb{R}$ will be called ***discretely harmonic***, if

$$f(x,y)=\frac{f(x+1,y)+f(x-1,y)+f(x,y+1)+f(x,y-1)}{4}.$$

Suppose we are given the values of a discretely harmonic function f at all even points except (0,0). Can we reconstruct $f(0,0)$?

3*. Prove that $x^{13}+2x^{12}-6x^{11}+2x^{10}-10x^8+4x^6+60x^5-44x^4-4x+4$ is irreducible over $\mathbb{Z}$.

4. You are given an N×M table of real numbers. The sum in every sub-square of 3×3 cells is positive, and the sum in any sub-square of 5×5 cells is negative. What can we claim about M and N?
In other words, for which M and N a table satisfying the conditions exists?

5*. Consider the polynomial $1+x^2y^4+x^4y^2-3x^2y^2$. Prove that it is non-negative for any real *x*, *y* and that this polynomial cannot be represented as a sum of squares of polynomials with real coefficients.