

## Targil 6 – discrete derivative.

1. Which functions satisfy the following condition: for every triple of different real numbers,  $x, y, z$ , the inequality  $\frac{f(x)}{(x-y)(x-z)} + \frac{f(y)}{(y-x)(y-z)} + \frac{f(z)}{(z-x)(z-y)} \geq 0$  holds?

2. A grasshopper performs an infinite sequence of jumps on the straight line. The length of the jump number  $n$  should be  $n^{5769}$ , but it is allowed to choose a direction of each jump. Show that it can visit all integer points if it wants.

3. Is it possible to divide  $[0, 1]$  into black and white intervals so that for each polynomial of degree  $< 5769$ , we shall have  $\int_{white} p(x)dx = \int_{black} p(x)dx$  ?

4. a. Show that each integer number can be written as a sum of 5 cubes of integer numbers.

b. Find some natural number  $N$  (as small as You can), such that each integer number is a sum of  $N$  numbers of type  $k^{2009}$ , for integer  $k$ .

5\*. Consider a set of points  $(x, y, z)$  such that  $x, y, z$  are integer nonnegative numbers not bigger than  $n$ , which cannot be simultaneously 0. What is the minimal number of planes not passing through  $(0, 0, 0)$  that contain all those points?