

Targil 5 – double counting.

1. In one country, there are 5 big and 19 small cities. The country is divided into 9 regions. Each big city is connected by bus to at least 14 cities, while each small city is connected by bus to at most 3 cities (each bus goes in both directions). Show that there exists a region in which no two cities are connected by bus.

2. a. Show that each map on a sphere has a country with less than 6 neighbors, and conclude that each map can be painted in 6 colors, so that countries having common border of positive length will be of different colors.

b. Show that each map on a sphere can be painted in 5 colors.

c. What is the maximal number of necessary colors for a map on a torus?

3. There are l unit vectors in n -dimensional space which are pair-wise orthogonal, and the orthogonal projection of each vector to a given k -dimensional subspace is longer than ε . Show that $l \leq k / \varepsilon^2$.

4. In a table there are N columns and M rows, $N > M$.

Some cells are marked by stars, and in each column there's at least one star.

Show that there is a star for which there are less stars in its column than in its row.

[Another way to formulate essentially the same problem: the books in the library were rearranged, so that for each book we have more books on the same shelf with it than before; show that now there is an empty shelf].

5*. Show that in a group of 50 people there are two that have an even number of common friends (maybe 0), assuming that friendship is symmetric.