Targil 4 – parity and divisibility.

1. We are given a herd 2009 cows. For each cow, if it is taken aside, others can be divided into two sub-herds of 1004 cows and equal total weight.
Prove that all the cows have the same weight.

2.** (a) A square is divided into N triangles of equal area. Prove that N is even.
(b) Generalize it for higher dimensions (a cube is divided into simplexes).

3. On infinite empty chessboard, a rectangle of $m \times n$ pieces is placed. One type of operation is allowed: a piece can jump above the piece in adjacent cell to the next cell after it, which should be free, and then the piece above which it jumped is removed.
By adjacent cells we mean cells with common side.
The purpose of the game is to leave only one piece on the board.
For which m, n is it possible?

4. Two players play a game on the standard empty chessboard.
They have a chess knight (horse). The first player places it on the chessboard at any cell he wishes, then the second makes a legal move with the knight, then the first makes a legal move and so on. In addition to standard chess rules, the knight is forbidden to step on the same cell twice.
The player that can't make a move in his turn loses. Who of the two players has a chance to win?

5*. Let T be the set of all numbers of the form m^n, where $m > 1$ and $n > 1$ are integer. Compute $\sum_{t \in T} \frac{1}{t-1}$.

(Since T is a set, a number which can be represented both as m^n and a^b is counted only once.)