

Targil 2 - some linear algebra.

1. Let R be a 3×3 matrix representing rotation of Euclidean space. How to compute the angle of rotation? And the axis?

2. Assume $\alpha \neq 0$ is a real number and F, G are two linear maps (operators) on \mathbb{R}^n such that $FG - GF = \alpha F$.

(a) Prove that $F^k G - GF^k = \alpha k F^k$.

(b) Prove that $F^k = 0$ for certain k .

3. (a) Is it true that for each couple of square matrices A, B , matrices AB, BA are similar?

(b) Is it true that A and A^T are always similar?

(Reminder: matrices X and Y are similar iff $X = PYP^{-1}$ for some invertible P , that means, the matrices represent the same linear transformation in a certain basis.)

4*. (a) Let $A_1 A_2 \dots A_n$ be a regular polygon, O its center. For any point X , consider perpendiculars from X to the lines of sides of the polygons as vectors starting at X and ending on corresponding sides. Prove that sum of those vectors is $nXO/2$.

(b) In similar problem in a platonic solid of n faces, the answer is $nXO/3$.

5**. Consider an anti-symmetric ($A = -A^T$) matrix with integer coefficients. Show that the determinant is a perfect square.