

Targil 12 – Analytic Geometry.

1. Consider segments AB, such that A is on x axis, B is on y axis, and length of AB is 1. The union of these intervals is a planar shape. Find an equation of the boundary of that shape.

2. For each t , take a line going through two points: $(t, 0)$ and $(0, 1 - t)$. When we draw all these line, part of the plane will be painted. Find a curve that separates the painted part of the plain from the unpainted.

3.* We are given an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. A circle with center O is tangent to the ellipse externally (meaning they don't have internal common point); at the same time, there are two parallel lines tangent to both the circle and the ellipse. Find the locus of O satisfying these conditions.

4.* Let P be a point upon the rectangular hyperbola $\{xy = 1\}$. Let D be a symmetric point to P with respect to 0. Suppose a circle with center at P intersects the hyperbola $\{xy = 1\}$ at 4 points: A, B, C, D. Prove that ABC is equilateral (regular) triangle.

Reminder. Each hyperbola has two asymptotes – straight lines that approximate it very well at all distant points. Hyperbola is called rectangular, if the asymptotes are orthogonal.

5.** For a triangle ABC in plane, consider rectangular hyperbolas, going through A, B and C simultaneously. Each of those hyperbolas has a center of symmetry. Prove that all these centers lie on one circle.

6. ABCD is a tetrahedron in the space. For each edge, consider plane passing via its midpoint and orthogonal to the opposite edge (for instance, a plane via the middle of AB orthogonal to CD). Prove that these 6 planes intersect in one point.