Targil 10 –algorithms

1. A road in desert area is a real line with camps at integer points. It is a day's walk between two camps. A human can carry 3 packed lunches, while he or she consumes one lunch each day. There is a base at 0. Things can be left only in the camps. It is required to organize an expedition, which will leave a packed lunch at camp 5, and all members of the expedition should return alive to the base. How many packed lunches are required?

2. (a) A soldier is a finite automata: his head has a finite number of states, and he can respond to a finite number of commands. A row consists of N soldiers, which are in the same initial state. Prove that they can be programmed so that several seconds after the leftmost soldier will receive a specific command, they will shoot simultaneously. Each soldier can pass a command to every neighbor during any second.
(b)* Show that for a row of length N no more and no less than 2N – 2 seconds are needed between the first command and the shooting, if the most efficient algorithm is used.

3. Show that in Conway's game "Life" there is a configuration without pre-image.

(The Game of Life http://en.wikipedia.org/wiki/Conway's_Game_of_Life is played on an infinite two-dimensional grid of square *cells*, each of which is in one of two possible states, *live* or *dead*. Every cell interacts with its eight *neighbors*. At each step in time, the following transitions occur:
1. Any live cell with fewer than two live neighbors dies, as if caused by underpopulation.
2. Any live cell with more than three live neighbors dies, as if by overcrowding.
3. Any live cell with two or three live neighbors lives on to the next generation.
4. Any dead cell with exactly three live neighbors becomes a live cell.)

4.* Denote C(n) minimal number of operations required to multiply a segment by n using compass. Denote CR(n) minimal number of operations required to multiply a segment by n using compass and ruler. Prove that C(n)/CR(n) is unbounded.

5.** An infinitely wise but a shortsighted cockroach is trying to find the truth (on the Euclidean plane). If he is in a distance of less than one step from the truth, he will reach it with the next step. After each step he is told whether he got closer to the truth. In the beginning he knows, that the truth is N steps from him.
Prove that the minimal number of steps required to find the truth is
    (a) Less than $N + 10 \log_2(N)$
    (b) More than $N + \log_2(N)/10$