Targil 1 - polynomials.

1. A polynomial $p(x)$ of degree n has only integer values in integer points.
(a) Show that $n!p(x)$ has integer coefficients.
(b) Can we claim that $p(x)$ has integer coefficients?

2. Let $p(x)$ be a polynomial with integer coefficients, and $a_1 < a_2 < \ldots < a_n$ integer numbers.
(a) Prove that there always exists an integer a such that $p(a)$ is divisible by $p(a_1)$, $p(a_2)$, … , $p(a_n)$.
(b) Can we claim that there always exists an integer a such that $p(a)$ is divisible by $p(a_1)p(a_2)\cdot \ldots \cdot p(a_n)$?

3. Let $P(x)$ be polynomial with integer coefficients of degree $n > 1$.
Consider a polynomial $Q(x) = P(P(P(\ldots P(P(x))\ldots)))$, where P occurs n times.
Show that Q has no more than n integer stable points, i. e. no more than n integers such that $Q(z) = z$.

4. Consider a graph of a polynomial $p(x)$ of degree n on a plane, and a point P on the same plane. Show that there are no more than n tangent lines to the graph of $p(x)$ passing through P.

5*. Prove that $5765^{5765}+5766$
(a) is not a prime number
(b) is a product of three numbers which are greater than 1.