Selection of Israeli Team for IMC 2008.

1. A group is generated by two elements a,b. The following relations hold:

$$a^2 = 1$$
$$b^2 = 1$$
$$(ab)^{10} = 1$$

Find the maximal possible size of this group.

2. For $f : \mathbb{R} \to \mathbb{R}$ it is given that any positive real x, y the sequence $f(x+ny)$, for $n \in \mathbb{N}$, tends to infinity. Does it follow that $f(x) \to \infty$ as $x \to \infty$?

3. Denote $[X,Y] = XY - YX$ (it is called commutator). Assuming that A, B, C are 2×2 matrices, prove that $\left[[A,B]^2, C\right] = 0$.

4. For which N can we draw a full graph of N vertices on the plane, so that each arc will be intersected no more than once, and no three arcs would have a common inner point?
(A graph is called ***full***, if each two vertices are connected by an arc.)

5. For tetrahedron ABCD, an altitude (גובה) is a straight line passing trough one vertex and orthogonal to the plane containing 3 other vertexes.
It is given, that no two edges of ABCD are orthogonal.
Prove that there exists a straight line, passing through vertex A and having a common point with each tetrahedron's altitude.

Good luck!