Targil 9 (following Alexey's story, about catastrophes and linear variations)

1. Given a polynomial of degree 3 with real coefficients: $x^3 + ax + b$.
Prove that it has 3 distinct real roots iff $(a/3)^3+(b/2)^2$ is negative;
and that it has only one real root of multiplicity one iff $(a/3)^3+(b/2)^2$ is positive.

2. In Moscow they have 9 sky-scrapers (assume Moscow is a plane, and sky-scrapers are points). A tourist, that stands at a certain point and looks around (counterclockwise), will see them in a certain cyclical order, at least if he doesn't stand on a line connecting two sky-scrapers. There might be 8! different cyclic orders.
(a) is it possible that each order will appear at some point?
(b) for which minimal number of buildings, will it be possible to see the buildings in every cyclic order?

3. A convex body C is contained by the unit cube. Projection of C to every face of the cube cover it completely. What is the minimal possible volume of C?

4. A rectangle is divided into disjoint union of the finite number of squares. Prove, that the aspect ratio (width / height) of the rectangle is a rational.

5. A square matrix is called ***bi-stochastic*** if all its numbers are nonnegative and sum of numbers in each column and in each row is one.
Prove that any bi-stochastic matrix is a linear combination of permutation matrix, with positive coefficients.