Targil 8: polytops.

1. Compute: an angle between faces of a regular tetrahedron, an angle between the adjacent faces of a regular octahedron, and an angle between the two different long diagonals (connecting pairs of opposite vertices) in a cube.

2. (a) Tetrahedron A (not necessary regular) contains strictly tetrahedron B.
Can we claim that the sum of lengths of all edges of A is bigger then the sum of lengths of all edges of B?
(b) Can we prove a similar statement if we would take rectangular parallelepipeds (= cuboids = boxes) instead of tetrahedron?
(c) Can we prove a similar statement if we would take sums of areas of faces instead of sums of lengths of edges both in (a) and (b) ?

3. Is it possible to inscribe a regular octahedron in a cube, so that all octahedron's vertices will be inside cube's edges?

4. Given a unit cube, a line, and a plane orthogonal to the line. Prove that the length of the projection of the cube on the line is equal to the area of the projection of the cube on the plane (of course, line is not necessarily parallel to one of the cubes edges).

5. A dodecahedron and an icosahedron (both regular) have a common circumsphere (means they are inscribed in the same sphere). Prove that they have a common insphere (means there is a sphere tangent to all the faces of both polytops).

6. We are given 100 vectors $\{(x_k , y_k , z_k)\}$ such that $-1 \le x_k , y_k , z_k \le 1$ for all k.
A sum of all this vectors is considered, while we are allowed to change signs in certain vectors. We change the signs in such a way, that the result of this sum will be as short as possible.
(a) Show that we can choose the signs so that the length of the sum will be ≤ 3.
(b) Show that the previous statement would be wrong if we would take 2 instead of 3.
***(c) Find the minimal value, for which (a) will hold.