Targil 5. Combinatorics again, but now with infinite sets.

1. Show that each sequence $\{a_n\}$ of real numbers has either infinite non-decreasing subsequence or infinite non-increasing subsequence.

2. Consider a set of distinct points in space $\{(x_i, y_i, z_i)\}$ such that all their coordinates are natural (positive integers). (Not all points of that kind, just some of them.)
Point (x_i, y_i, z_i) of this set is called minimal if for every other point (x_k, y_k, z_k) in this set, point has a smaller coordinate $x_i < x_k$, or $y_k < y_i$, or $z_i < z_k$.
Can a number of minimal points be infinite?

3. Show that there is a point in the plane, such that distances from it to all integer points are different. (A point (x,y) is called integer if x and y are integer).

4. (a) Is it possible to find 1000000 points in the plane, not all of them on one line, so that the distance between each two is integer?
(b) Is it possible to choose an infinite set of points in the plane, not all of them on one line, so that the distance between each 2 is integer?

5*. (a) We have a family of subsets of a countable set (say, natural numbers).
Each two members of the family have no more than 100 common elements.
Prove that the family is of countable size (at most).
(b) We have a family of subsets of a countable set, such that intersection of each 2 is finite. Can this family have more than a countable number of elements?