Targil 4. This time – combinatorics.

1. Six singers came to a festival. Each day several singers make a performance at the scene, while others listen from the audience.
How to organize the shortest possible festival, so that each singer will listen to every other singer?

2. Once upon a time, 17 cannibals have gathered together for a feast. During the feast, some cannibals were killed and eaten by some other cannibals.
It is known however, that there were no 5 cannibals such that neither of them has eaten any of the other 4.
Prove that there is a chain of 5 cannibals, such that each (except the 5^{th}) was eaten by the next member of the chain.

3. (a) A graph of 10 vertices doesn't contain 3 vertices which are all adjacent to each other. How many edges (at most) can it have?
(b) A graph of 10 vertices doesn't contain 4 vertices which are all adjacent to each other. How many edges (at most) can it have?

4. Yoav, when he was at school, once participated in a test in Arabic language (mevhan bearavit). During the test, he should have connected 10 Hebrew words to their 10 Arabic translations. He didn't know Arabic at all, so he has chosen the translation at random. His mark was 0, which means he had no correct guesses.
Many years later he asked me a question, and it is also my question to you. What was a probability of getting 0 at that test?

5.** There is a long line of $m+n$ people, which all want to buy an ice-cream. The price is 5₪, and out of those people m have a coin of 5₪, and n have a coin of 10₪, and the stand in random order. In the beginning, the cashier has k coins of 5₪. What is the probability that they won't get stuck?

6.** A rectangle is called ***good***, if one of its sides is integer. A big rectangle is divided into a finite number of smaller rectangles, which are all good. Show that the big rectangle is also good.