This time problems are about factorials.

1. (a) Prove that $\int_0^\infty e^{-x} x^N dx = N!$ for any natural *N*.

(b) Prove that $\int_0^1 x^K (1-x)^M \, dx = \frac{K!M!}{(K+M+1)!}$ for natural *K*, *M*.

2. Define (using 1a) and compute ½!

3. Compute the volume of the N-dimensional ball of radius 1 and the area of its surface.

4*. (a) Let $I_N = \int_0^{2\pi} \sin^N(x) dx$. Compute I_N for each integer *N*.

(b) Show that $\sqrt{N\pi} < 4^N \Big/ \binom{2N}{N} < \sqrt{\left(N+\frac{1}{2}\right)\pi}$

5. Compute $\lim_{N\to\infty} \left(\frac{\sqrt[N]{N!}}{N} \right)$.