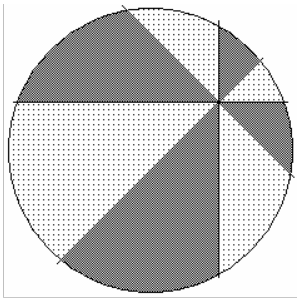
This time the problems are about differentiating/integrating in geometrical context.

1. Consider a sphere (the surface of a ball) of radius *R*, and cut it by a horizontal plane at distance *h* from its north pole. Compute the area of the part of the sphere which is above the plane (spherical kipa).



2. A circle was cut by 4 lines, passing through the same interior point of the circle and having forming 8 equal angles at that point (of 45 degrees each).
So, the inner part of the circle is divided into 8 parts, which are painted in 2 colors, so that neighboring parts have different colors (like the chessboard).
Show, that each color takes the same part of the area.

3. Develop a formula for an area of a polygon, whose vertices are (x_1,y_1) (x_2,y_2), …, (x_n,y_n) (in this order).

4. Two convex polygons P, Q are given, such that Q is strictly inside P.
A point X on the boundary of Q is called good, if there are two points A, B on the boundary of polygon P such that line AB doesn't cut the polygon Q in two parts, and X is a middle point of the interval AB.
Show that there are at least 2 good points.

5. Consider a billiard ball on a table bounded by a convex smooth wall without pockets. The ball is so small that we consider it to be a point. The trajectory of the ball is a straight line, until it hits the wall, at the wall it is reflected so that the angle between the wall and the trajectory is preserved. So the trajectory is a broken line, consisting on intervals.

A convex line inside the table is called *caustics*, if the following property holds:
If the trajectory of the billiard ball touches this line once, then it touches it on with interval.
We are given a smooth convex line inside the billiard table which is caustics.
For each point A of the edge of the table, let p(A) be the perimeter of the convex hull of A and the given caustics.
Show that p(A) doesn't depend on A.