Targil 13 – limits.

1. Compute $\lim_{x\to 0+}\int_{x}^{2x}\frac{\sin^m(t)}{t^n}dt$, where m and n are natural numbers, (the answer may depend on m and n).

2. a) Does $\sum_{k=0}^{\infty}\frac{1}{k\cdot(\ln k)\cdot(\ln(\ln k))}$ converge?

b) Does $\sum_{k=0}^{\infty}\frac{1}{k\cdot(\ln k)\cdot(\ln(\ln k))^2}$ converge?

3. Let $\{a_n\}$ be a sequence defined by $a_0 = 1$ and recursive formula

$$a_{n+1}=\frac{1}{n+1}\sum_{k=0}^{n}\frac{a_k}{n-k+2}$$

Find the limit $\sum_{k=0}^{\infty}\frac{a_k}{2^k}$.

4*. Compute $\lim_{x\to 0}\frac{\sin(\tan(x))-\tan(\sin(x))}{\arcsin(\arctan(x))-\arctan(\arcsin(x))}$.

5*. $\{a_n\}$ is a sequence of positive real numbers such that $a_n + a_m \geq a_{n+m}$ for all m, n.
Prove that the sequence a_n/n converges.