These problems are about determinants and linear algebra.

1. Compute $\left\|\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 6 & 10 & 15 \\ 1 & 4 & 10 & 20 & 35 \\ 1 & 5 & 15 & 35 & 70 \end{pmatrix}\right\|$

(First row/column consist of 1's, any number which is neither in the first row nor in the first column, is sum of its neighbors from above and from the left).

2. Consider two quadratic polynomials, ax^2+bx+c, dx^2+ex+f, where $a,d \neq 0$. Prove that they have the common root if and only if the matrix

$$\begin{pmatrix} a & b & c & 0 \\ 0 & a & b & c \\ d & e & f & 0 \\ 0 & d & e & f \end{pmatrix}$$

is degenerate.

3. A quadric in plane is a locus of zeroes of an equation of order 2:
$ax^2+bxy+cy^2+dx+ey+f=0$. (At least one coefficient should be nonzero)
a. Show that for each 5 points there exists a quadric containing all of them.
b. Show that if two quadrics have only finite number of common points,
than it isn't bigger than 4.

4. Find the roots of the polynomials:

a. $\left\|\begin{pmatrix} x & 1 & 0 & 0 & 1 \\ 1 & x & 1 & 0 & 0 \\ 0 & 1 & x & 1 & 0 \\ 0 & 0 & 1 & x & 1 \\ 1 & 0 & 0 & 1 & x \end{pmatrix}\right\|$ b. $\left\|\begin{pmatrix} x & 1 & 0 & 0 & 0 \\ 1 & x & 1 & 0 & 0 \\ 0 & 1 & x & 1 & 0 \\ 0 & 0 & 1 & x & 1 \\ 0 & 0 & 0 & 1 & x \end{pmatrix}\right\|$

5. Write an equation which holds if and only if the four points
(x_1,y_1), (x_2,y_2), (x_3,y_3) (x_4,y_4) lie on one circle or one straight line.